

A DUALITY THEOREM FOR GRAPHS, CODES, AND MATROIDS

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Graphs

Let G be a multigraph with n edges.

Define

k = # of edges in a spanning forest of G

b_i = minimal # of edges in a union of i bonds, none contained in the union of the others

c_j = minimal # of edges in a union of j cycles, none contained in the union of the others.

Set

$$U = \{b_1, \dots, b_k\}$$

$$V = \{n + 1 - c_{n-k}, \dots, n + 1 - c_1\}.$$

Codes

Let C be an $[n, k]$ linear code over some field.

Define for $\mathbf{v} \in C$ and $D \subseteq C$

$$\chi(\mathbf{v}) = \{i : v_i \neq 0\} \quad \chi(D) = \bigcup_{\mathbf{v} \in D} \chi(\mathbf{v})$$

$$w(D) = |\chi(D)|$$

$$d_i = \min\{w(D) : D \text{ an } [n, i] \text{ subcode of } C\}$$

$$d_j^\perp = \min\{w(D) : D \text{ an } [n, j] \text{ subcode of } C^\perp\}.$$

Set

$$U = \{d_1, \dots, d_k\}$$

$$V = \{n + 1 - d_{n-k}^\perp, \dots, n + 1 - d_1^\perp\}.$$

Matroids

Let M be a matroid of rank k on n elements.

Define

f_i = maximal size of an i -rank set in M

f_j^* = maximal size of an j -rank set in M^* .

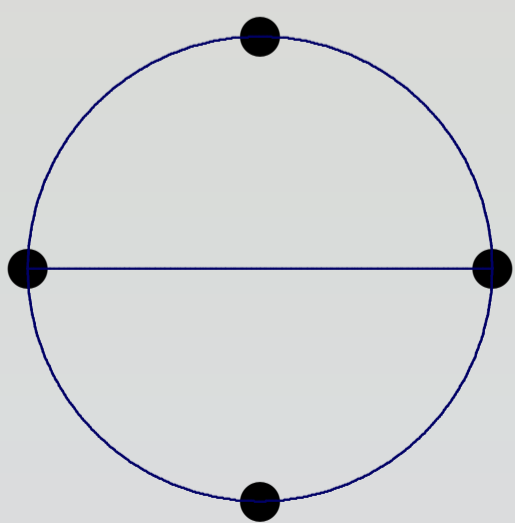
Set

$$U = \{f_0 + 1, \dots, f_{k-1} + 1\}$$

$$V = \{n - f_{n-k-1}^*, \dots, n - f_0^*\}.$$

Theorem : $U \cup V = \{1, \dots, n\}$ and $U \cap V = \emptyset$.

Example



$$n = 5 \quad b_1 = 2 \quad U = \{b_1, b_2, b_3\} = \{2, 4, 5\}$$

$$k = 3 \quad b_2 = 4 \quad V = \{5 + 1 - c_2, 5 + 1 - c_1\} = \{1, 3\}$$

$$b_3 = 5$$

$$c_1 = 3$$

$$c_2 = 5$$

$$U \cup V = \{1, 2, 3, 4, 5\} \text{ and } U \cap V = \emptyset$$

References

The code and graph theorems first appeared in

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